

Thermal hybrid inflation in brane world

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Abstract

In conventional scenario of thermal inflation, the requirement that the reheating temperature must be larger than the temperature of the nucleosynthesis puts a lower bound on the mass of the inflaton field. At the same time, the mass of the inflaton field and the height of the potential during thermal inflation are intimately related. With these conditions, the conventional models for thermal inflation are quite restricted. Naively, one can expect that the above constraints may be removed if thermal inflation is realized within the setups for hybrid inflation. In this paper we show why it is difficult to construct hybrid models for thermal inflation within conventional supergravity, and then show a successful example in models for the braneworld. Our mechanism is based on the idea of non-tachyonic brane inflation.

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1 Introduction

In conventional models for thermal inflation, inflation starts from the state where the symmetry is restored by the thermal effect. One may regard it as a modification of new inflation, where inflation starts at the top of the potential. On the other hand, hybrid inflation starts with large field initial condition. During hybrid inflation, the inflaton field stabilizes the trigger field at the false vacuum. In this respect, hybrid inflation is similar to chaotic inflation. Taking these things into consideration, it seems rather difficult to construct thermal hybrid inflation without adding unnatural components by hand. What we want to show in this paper is that the collaboration between these different kinds of inflation can be realized in the natural settings of brane inflation.²

2 Thermal hybrid inflation

In this section we first consider an old idea of inverted hybrid inflation[3] where the hybrid potential is used while the initial field expectation value is not large.

Inverted hybrid inflation

In the scenario for inverted inflation, the inflaton field slowly rolls away from the origin, and finally the trigger field terminates inflation. Thus at first sight, it seems possible to construct thermal hybrid inflation, if it is realized in the setups of the inverted scenario. Here we examine the above simplest idea and show that it is indeed quite difficult to be realized without fine-tunings.

Perhaps the simplest way to explain the idea of inverted hybrid inflation is to consider the potential

$$V(\phi, \sigma) = \frac{1}{4}\lambda_\sigma (\sigma^2 + M^2)^2 - \frac{1}{2}\lambda_{int}\phi^2\sigma^2 + \frac{1}{4}\lambda_\phi (\phi^2 - M'^2)^2 \quad (2.1)$$

where ϕ is the inflation field and σ is the trigger field. Inverted hybrid inflation starts when the inflaton field (ϕ) stays at the top of the potential ($\phi \simeq 0$), where the trigger field is stabilized at $\sigma = 0$. Here we examine if the thermal initial condition works well for this model. We assume that, because of the thermal effect, ϕ is held at the origin during

²In general, inflation with low fundamental scale is very difficult[1]. Baryogenesis in models with low fundamental scale is discussed in ref.[2].

thermal inflation. Since the inflaton field (ϕ) stays at $\phi = 0$, the trigger field (σ) is also held at a false vacuum $\sigma = 0$ until ϕ reaches the critical value ϕ_c . Then the effective mass squared for the σ field becomes negative, allowing σ to roll down to its true vacuum. In this simplest model, however, the dimensionless coupling constant λ_ϕ must be fine-tuned so that thermal inflation is allowed, which is the same situation as the original model for thermal inflation[7]. Naively, one may consider a flat potential for ϕ

$$V(\phi, \sigma) = \frac{1}{4}\lambda_\sigma (\sigma^2 + M^2)^2 - \frac{1}{2}\lambda_{int}\phi^2\sigma^2 - \frac{1}{2}m_\phi^2\phi^2 + A_n\frac{\phi^{n+4}}{M^n}, \quad (2.2)$$

assuming that the potential for the inflaton ϕ is flat in the supersymmetric limit and is destabilized by the soft supersymmetry breaking term, which is denoted by $-\frac{1}{2}m_\phi^2\phi^2$. In this case, the flatness of the potential seems to be ensured by supersymmetry. However, as is already discussed in ref.[4], it is hard to obtain a flat potential while making the required coupling large enough to destabilize the trigger field at the end of inflation. Moreover, if one wants to construct “strong” thermal inflation with the number of e-foldings $N_e > 35$, one should consider another constraint for $\lambda_\sigma M^4$. During thermal inflation, the inflaton field is stabilized at the symmetric point by the thermal effect. Thermal inflation starts at $T_{in} \simeq \left(\frac{1}{4}\lambda_\sigma M^4\right)^{1/4}$ and ends at $T_c \simeq m_\phi$. The expansion during this period is $N_e \simeq \ln\left(\frac{T_{in}}{T_c}\right)$. In order to obtain large number of e-foldings, the vacuum energy during thermal inflation must be as large as $V_0^{1/4} \simeq m_\phi e^{N_e}$. Within the setups of the conventional models for supergravity, it seems very hard to satisfy any of the above constraints without adding unnatural extra components.

Thus we conclude that the model is not suitable for our purposes. We will show that these problems are solved in the setups of brane inflation.

Non-tachyonic brane inflation

Here we consider non-tachyonic brane inflation in ref.[5], and examine whether one can construct a model that is suitable for our purposes. The model should be similar to the model of inverted hybrid inflation that we have discussed above. For example, we consider a potential of the form

$$V(\phi, \sigma) = \frac{1}{4}\lambda_\sigma (\sigma^2 + M^2)^2 - \frac{1}{2}\lambda_{int}e^{-(M_0 r)^2}\phi^2\sigma^2 + \frac{1}{4}\lambda_\phi (\phi^2 - M'^2)^2. \quad (2.3)$$

Here we have considered two branes at a distance, which we denote by 1 and 2. The field

σ and the field ϕ are localized on brane 1 and brane 2, respectively.³ The interaction term is accompanied by an exponential factor, because the field ϕ and the field σ are localized on different branes at a distance r . One can easily find that the above effective four-dimensional potential is a simple modification of the potential for inverted hybrid inflation. In this model, the inflaton field is the moduli for the brane distance r , which we denote by $\psi = M_*^2 r$. The most obvious difference is that the field ϕ is not required to be placed at the unstable point. Inflation starts because the exponential factor is nearly zero at the beginning of inflation. If $\lambda_{int} M'^2 > \lambda_\sigma M^2$, the field σ is destabilized at the end of inflation, when two branes come close.

It is possible to express the above idea (separation between branes drives hybrid inflation) in the explicit supersymmetric form[5]. The simplest example is the F-term brane inflation. On one brane, we assume a localized superfield Σ and its superpotential of the form

$$W_1 = \Sigma M^2. \quad (2.5)$$

On the other brane, we assume that another field Φ is localized and its superpotential is $W_2 = 0$. When two branes come close, we assume another superpotential of the form

$$W_{int} = \Sigma(M^2 - \lambda_i \Phi^2 e^{-(M_0 r)^2}), \quad (2.6)$$

where r denotes the distance between branes. One can explain the forms of the superpotential by imposing R-symmetry. In eq.(2.3), the vacuum energy during inflation is positive because σ is located at the false vacuum ($\sigma = 0$). In the present case, the vacuum energy during inflation is due to the supersymmetry breaking induced by Σ . In both cases, the interaction is restored at the end of inflation. In eq.(2.3), the potential for σ is destabilized by the interaction. In the present case, supersymmetry restoration is induced

³Here we consider the case in which fields are localized on the branes. The forms of their wave-functions will be gaussian that decays exponentially in the transverse directions. For example, we consider the form;

$$\phi(r) \sim e^{-(M_0 r)^2}, \quad (2.4)$$

where r denotes the transverse distance from the center of the brane, and M_0^{-1} is the width of the wave-function. When branes are separated, the couplings between localized modes from different branes will decay exponentially. On the other hand, when branes are on top of each other, the couplings are restored. The effect of the localized wave-functions is included in the second term in eq.(2.3).

by the interaction term.

For an another example, we may consider a localized Fayet-Iliopoulos D-term on a brane at $\vec{r} = 0$,

$$\xi D \delta(\vec{r}) \quad (2.7)$$

where D is an auxiliary field of the vector superfield. We consider an additional abelian gauge group $U(1)_X$ in the bulk, while the Fayet-Iliopoulos term for $U(1)_X$ is localized on a brane. We also include the field ϕ_X that has $U(1)_X$ charge and localized on the other brane at $\vec{r} = \vec{r}_1$. Then the D-flat condition is

$$\xi \delta(\vec{r}) + c_X (|\phi_X^+|^2 - |\phi_X^-|^2) \delta(\vec{r} - \vec{r}_1) = 0, \quad (2.8)$$

where c_X is a constant, and the δ -functions are gaussian located on each branes. When two branes are placed at a distance ($|\vec{r}_1| \gg M_*^{-1}$), the Fayet-Iliopoulos term breaks supersymmetry on the brane, and inflation starts. In this case, as in the conventional models for brane inflation, the inflaton field is the moduli that parametrizes the brane distance. The moduli is denoted by $\psi = M_*^2 r_1$, where M_* denotes the fundamental scale of the model.

The issue of the potential for the brane distance is discussed by many authors for many models. Of course, there are many contributions at the same time, which will take different forms. For the typical example,

- The simple $m_{3/2}^2$ correction from the effective low energy discription of supergravity.
- The loop corrections to the Kähler metric that comes from the particles of the mass $\sim M r^2$ [6].
- Since the cancellation between the graviton-dilaton attraction and the RR repulsion fails when supersymmetry is broken, the Van der Waals forces between branes will appear [6].

The trigger field is the localized field ϕ_X , which develops vacuum expansion value to compensate the D-term (2.7) when two branes come close.

The most significant difference from the original model for inverted hybrid inflation is that the interaction that destabilizes the trigger field is accompanied by an exponential

factor $e^{-(M_*^{-1}\psi)^2}$. The destabilization in the potential (2.3) is not due to the variation of the field ϕ , but due to the variation of the effective interaction constant $\lambda_{int}e^{-(M_*^{-1}\psi)^2}$.

In this case, however, the initial expectation value for the inflaton is large. In this respect, to collaborate with thermal inflation, we should modify the initial condition for the above extended model for hybrid inflation. To find the solution for the problem, we must first review the original idea for thermal brane inflation that was advocated in ref.[6]. In ref.[6], it is discussed that an open string model can be thermalized to stabilize a brane on the top of the different brane. Unlike the usual models for brane inflation that starts with a large expectation value for the inflaton field, thermal brane inflation starts at the top of an another brane.

Thermal brane inflation (original)

Let us briefly review the idea of thermal brane inflation proposed by Dvali[6]. The following conditions are required so that the mechanism functions.

1) Exchange of the bulk modes such as graviton, dilaton or RR fields govern the brane interaction at the large distance.

2) In the case when branes initially come close, bulk modes are in equilibrium and their contribution to the free energy can create a positive T^2 mass term for ψ to stabilize the branes on top of each other until the Universe cools down to a certain critical temperature $T_c \sim m_s$.⁴ Here m_s represents the negative curvature of ψ at the origin, which is determined by the supersymmetry breaking.

The resultant scenario of thermal inflation is straightforward. Assuming that there was a period of an early inflation with a reheat temperature $T_R \sim M$, and at the end of inflation some of the repelling branes sit on top of each other stabilized by the thermal

⁴The author of ref.[6] considered open string modes stretched between different branes. When branes are on top of each other, these string modes are in equilibrium and their contribution to the free energy creates a positive T^2 mass term. One may also worry about the problem of the domain wall formation. In this model, domain wall formation is possible only when the broken symmetry allows cosmological domain walls. For example, domain walls may be formed when the number of the transverse dimension is one, and at the same time the distant brane B is not located at the boundary. In such a case, the moving brane will have degenerated \pm directions. The above situation is already excluded in the original model[6].

effects, one can obtain the number of e-foldings

$$N_e = \ln\left(\frac{T_R}{T_c}\right). \quad (2.9)$$

Taking $T_R \sim 10\text{TeV}$ and $T_c \sim 10^3 - 10\text{ MeV}$, one finds $N_e \sim 10 - 15$, which is consistent with the original thermal inflation[7] and is enough to get rid of unwanted relics.⁵ In the original model the crucial restriction appears in m_s , which must be large enough to satisfy the lower limit for the reheating temperature.⁶ In our model, the above restriction is actually removed.

Hybrid alternative (Thermal hybrid inflation in the brane universe)

Now it seems straightforward to improve the initial condition for non-tachyonic brane inflation to fit the settings of thermal brane inflation. To explain the idea, let us consider two branes (brane A and brane B) fixed on some point in the extra dimensions, and a moving brane that is not fixed yet. In the true vacuum, the moving brane stays on top of brane A, while the thermal effects confine it on top of brane B during thermal inflation. As we have discussed above, the initial condition for the non-tachyonic brane inflation is satisfied within the above settings. What we want to consider in this paper is a hybridization of thermal brane inflation and non-tachyonic brane inflation. Thermal hybrid inflation occurs if a moving brane, which is responsible for the supersymmetry on brane A attaches to an another brane at a distance. If the components on each brane are appropriate to meet the requirement from non-tachyonic brane inflation, thermal inflation starts. At the end of thermal inflation, the moving brane falls apart from brane B and

⁵See also the previous arguments in ref.[8]

⁶For example, the reheating temperature for the original thermal brane inflation is[6]

$$T_R \sim \sqrt{\frac{m_{osc}^3}{\phi_0^2}} M_p \quad (2.10)$$

where m_{osc} and ϕ_0 are the mass of the oscillating inflaton field and the vacuum expectation value of the inflaton at the true vacuum, respectively. Here we have omitted the numerical factors. Of course, the negative mass at the origin (m_s) and the mass of the oscillating inflaton (m_{osc}) is different. However, as is discussed in the footnote 7, it seems hard to believe that $m_s \ll m_{osc}$ is realized in natural settings. Thus we assume $m_s > m_{osc}$. In any case, the reheating temperature must be larger than the temperature of the nucleosynthesis. On the other hand, if one requires large number of e-foldings, one needs small m_s and large ϕ_0 so that the thermal inflaton lasts long. In the original model for thermal brane inflaton, $m_{osc} \sim 10\text{MeV} \gg m_{3/2}$ is assumed.

moves toward brane A. When the moving brane come near to brane A, the spontaneously broken supersymmetry is recovered and the inflation ends with the oscillation of the field on the brane.

There are two unique characteristic features in this model. In conventional models for thermal inflation, there is a lower limit for the mass of the inflaton field that is derived from the requirement for the reheating temperature. This constraint is removed in our model because of the hybrid potential. The second is that thermal inflation can become “strong” in our model. To be more precise, we show why the conventional models for thermal inflation were “weak”. In generic models for thermal inflation, inflation starts at $T_{in} \simeq \sqrt{m_I M}$, where M and m_I are the vacuum explain value of the inflation field in the true vacuum and the mass of the inflaton field, respectively. Thermal inflaton ends at the temperature $T_{end} \simeq m_I$. During this period, the Universe expands with the number of e-foldings of

$$N_e \simeq \ln \left(\frac{T_{in}}{T_{end}} \right) \simeq \ln \left(\sqrt{\frac{M}{m_I}} \right). \quad (2.11)$$

In this case, even if M is as large as the GUT scale, the number of e-foldings is at most $N_e \simeq 17$ for $m_I \simeq 1\text{GeV}$. Thus we should conclude that the conventional model for thermal inflation is a model for weak inflation, which cannot be used for the first inflation.

In our model, however, the situation is changed. The energy density during inflation, which we denote by V_0 , is not related to the mass of the inflaton field. The expected number of e-foldings is

$$N_e \simeq \ln \left(\frac{T_{in}}{T_{end}} \right) = \ln \left(\frac{V_0^{1/4}}{m_I} \right). \quad (2.12)$$

Although F-term inflation is successful in our model, one must consider D-term inflation if one wants to protect inflaton mass from corrections of $O(H)$, where H denotes the Hubble parameter. For example, in models with $M_* > M_{GUT}$ and the gauge-mediated supersymmetry breaking on the brane, the number of e-foldings becomes $N_e \simeq 42$ for $V_0^{1/4} \simeq 10^{12}\text{GeV}$ and $m_I \simeq 1\text{keV}$. On the other hand, when the fundamental scale is low, one can obtain the number of e-foldings $N_e \simeq 35$ for $V_0^{1/4} \simeq 10^6\text{GeV}$ and $m_I \simeq 10^{-2}\text{keV}$.⁷

⁷Here we should explain how to get soft masses for the conventional supersymmetric standard model. In the above examples, we have assumed that the standard model fields and the source of the conventional supersymmetry breaking are localized on brane A. We have also assumed that the supersymmetry

3 Conclusions and Discussions

In this paper we have constructed an example for thermal hybrid inflation. In conventional thermal inflation, the requirement from the reheating temperature puts a lower bound on the mass of the inflaton field. The mass of the inflaton and the height of the potential during inflation is intimately related to prevent “strong” inflation. Our naive expectation is that one can remove the above constraints if thermal inflation is realized within the setups for hybrid inflation. With these things in mind, we have constructed a realistic model for thermal hybrid inflation. We have considered this problem in the scenarion of the brane world. Our result drastically modifies the cosmological scenarios related to thermal inflation. For example, in ref.[9], the curvaton hyposesis is discussed in the framework of thermal inflation. In ref.[9], they have concluded that the cosmological scales cannot leave the horizon during ordinary thermal inflation, because the constraint from the reheating temperature cannot meet the requirement. However, as we have discussed above, such a constraint is removed in our model.

We believe that our new model for thermal inflation opens up new possibilities for the brane world.

breaking is mediated to the standard model by renormalizable interactions on brane A. On the other hand, the source of the supersymmetry breaking during inflation can be located either on brane A or on the moving brane. Of course the scale of the supersymmetry breaking during inflation is higher and independent from the conventional (supersymmetric standard model) supersymmetry breaking. Considering the above conditions, we have assumed that the mass of the inflaton near brane B during inflation is mediated by the gravitational interactions. All the sources of the supersymmetry breaking are assumed to be located on brane A. If the source of the supersymmetry breaking is located on the moving brane, the inflation mass (m_I) during thermal inflation will become larger because during inflation the moving brane attaches to brane B, where excitation of light modes are expected[6].

The effective mass of the inflaton will be $m_I \simeq m_{3/2} \ll \sqrt{V_0/M_p}$, if thermal brane inflation is induced by the D-term.

In any case, the inflation mass (m_I) and the number of e-foldings (N_e) are strongly model dependent. We stress that m_I is not the mass in the true vacuum, but the effective mass at a distance.

4 Acknowledgment

We wish to thank K.Shima for encouragement, and our colleagues in Tokyo University for their kind hospitality.

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